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# Interacting six-dimensional topological field theories

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*Abstract.* We study the gauge-fixing and symmetries (BRST-invariance and vector supersymmetry) of various six-dimensional topological models involving Abelian or non-Abelian 2-form potentials.

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# 1 Introduction

Recently, L. Baulieu and P. West introduced a six-dimensional topological model of Witten-type involving 2-form potentials [1]. In the sequel, the gauge-fixing procedure and twist in this model have been studied in more detail in reference [2]; these authors also determined vector supersymmetry (VSUSY-) transformations which represent an additional symmetry of the model.

The goal of the present paper is to discuss two different generalizations of the free Abelian model [1] to interacting models. The first consists of coupling the Abelian 2-form potentials to a non-Abelian Yang-Mills field by virtue of a Chern-Simons term, as suggested in reference [1]. The corresponding action represents a six-dimensional topological version of the Chapline-Manton term appearing in the action for ten-dimensional supergravity coupled to super Yang-Mills theory [3]. The second generalization consists of considering non-Abelian (charged) 2-form potentials which are coupled to a Yang-Mills connection, following the lines of reference [4].

Before discussing these generalizations, we summarize the free Abelian model [1, 2] while using differential forms to simplify the notation (section 2). Our paper concludes with some comments concerning possible extensions of these six-dimensional topological models. We note that all of our considerations concern the classical theory (tree-level).

## 2 Free Abelian model

The arena is a compact pseudo-Riemannian 6-manifold  $\mathcal{M}_6$  and the basic fields are Abelian 2-form potentials  $B_2$  and  $B_2^c$  which are independent of each other. From the associated curvature 3-forms  $G_3 = dB_2$  and  $G_3^c = dB_2^c$ , one can construct the classical action [1]

$$\Sigma_{cl} = \int G_3 G_3^c. \quad (1)$$

Here and in the following, the integrals are understood as integrals of 6-forms over  $\mathcal{M}_6$  and the wedge product symbol is always omitted.

### 2.1 Symmetries

The action (1) is invariant under the *ordinary gauge transformations*

$$\delta_{\lambda_1} B_2 = d\lambda_1 \quad , \quad \delta_{\lambda_1} B_2^c = 0, \quad (2)$$

which represent a reducible symmetry in the present case, and it is invariant under the *shift-*(or topological Q-) *symmetry*

$$\delta_{\lambda_2} B_2 = \lambda_2 \quad , \quad \delta_{\lambda_2} B_2^c = 0, \quad (3)$$

which also represents a reducible symmetry. In equations (2),(3) and in the following, it is understood that the “ $c$ -conjugated” equations also hold, e.g.  $c$ -conjugation of equation (2) gives  $\delta_{\lambda_1^c} B_2^c = d\lambda_1^c$ ,  $\delta_{\lambda_1^c} B_2 = 0$ .

In the sequel, we will describe the infinitesimal symmetries in a BRST-framework and we will derive the BRST-transformations from a horizontality condition (Russian formula) [5]. Thus, we introduce a series of ghost fields associated with the reducible gauge transformations (2) and we collect them in a generalized 2-form,

$$\tilde{B}_2 = B_2 + V_1^1 + m^2. \quad (4)$$

Here, the upper and lower indices denote the ghost-number and form degree, respectively. The total degree of a field is the sum of its ghost-number and form degree, and all commutators  $[\cdot, \cdot]$  are graded with respect to this total degree. The BRST-differential  $s$ , which describes both the ordinary gauge transformations and the shift-transformations, is combined with the exterior derivative  $d$  in a single operator,

$$\tilde{d} = d + s, \quad (5)$$

which is nilpotent by virtue of the relations  $d^2 = s^2 = [s, d] = 0$ . Thus, the generalized field strength

$$\tilde{G}_3 \equiv \tilde{d}\tilde{B}_2 \quad (6)$$

satisfies the generalized Bianchi identity

$$\tilde{d}\tilde{G}_3 = 0. \quad (7)$$

The BRST-transformations of the classical and ghost fields<sup>1</sup> are now obtained from the *horizontality condition* [1, 2]

$$\tilde{G}_3 = G_3 + \psi_2^1 + \varphi_1^2 + \phi^3, \quad (8)$$

which involves a series of ghosts associated with the shift-symmetry. By inserting the field expansions (4) and (8) into relations (6) and (7), we obtain the  $s$ -variations [1, 2]

$$\begin{aligned} sB_2 &= \psi_2^1 - dV_1^1, & s\psi_2^1 &= -d\varphi_1^2 \\ sV_1^1 &= \varphi_1^2 - dm^2, & s\varphi_1^2 &= -d\phi^3 \\ sm^2 &= \phi^3, & s\phi^3 &= 0 \end{aligned} \quad (9)$$

and  $sG_3 = -d\psi_2^1$ . Since the field expansions (4) and (8) have not been truncated, the obtained BRST-transformations are nilpotent by construction [6].

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<sup>1</sup>Here, “classical” fields are not opposed to quantum fields, but simply refer to the fields appearing in the classical action.

## 2.2 Gauge-fixing

Let us briefly review the gauge-fixing procedure [1, 2] while using differential forms. To start with, we consider the shift degrees of freedom for the fields  $B_2$  and  $B_2^c$ . These are fixed by imposing a self-duality condition relating the corresponding field strengths:

$$*G_3 = -G_3^c. \quad (10)$$

This relation is equivalent to imposing a self-duality condition on  $G_- \equiv d(B_2 - B_2^c)$  and an anti-self-duality condition on  $G_+ \equiv d(B_2 + B_2^c)$ . Henceforth, relation (10) is analogous to the self-duality condition for the curvature 2-form  $F$  in four-dimensional topological Yang-Mills theory [7].

With the help of a BRST-doublet  $(\chi_3^{-1}, H_3)$ , i.e.

$$s\chi_3^{-1} = H_3 \quad , \quad sH_3 = 0, \quad (11)$$

the constraint (10) can be implemented in the gauge-fixing action:

$$\Sigma_{sd} = s \int \{ \chi_3^{-1} (*G_3 + G_3^c) \}. \quad (12)$$

Since the shift-symmetry represents a reducible symmetry, it is necessary to re-iterate the gauge-fixing procedure for the action  $\Sigma_{cl} + \Sigma_{sd}$ : this leads to the introduction of the anti-ghosts and Lagrange multipliers of tables 1 and 2, all of which have been arranged in Batalin-Vilkovisky pyramids. These fields again represent BRST-doublets:

$$\begin{aligned} s\phi^{-3} &= \eta^{-2} \quad , \quad s\eta^{-2} = 0 \\ s\varphi_1^{-2} &= \eta_1^{-1} \quad , \quad s\eta_1^{-1} = 0 \\ s\chi^1 &= \eta^2 \quad , \quad s\eta^2 = 0. \end{aligned} \quad (13)$$

In summary, the shift-invariance of the classical action is fixed by virtue of the gauge-fixing action

$$\Sigma_Q = \Sigma_{sd} + s \int \{ \varphi_1^{-2} d*\psi_2^1 + \phi^{-3} d*\varphi_1^2 + \chi^1 d*\varphi_1^{-2} - CC \}, \quad (14)$$

where  $CC$  stands for the  $c$ -conjugated expressions.

The reducible gauge symmetry (2) is fixed in a similar way: one considers the usual gauge condition  $d*B_2 = 0$  and re-iterates the gauge-fixing procedure. This leads to the introduction of the series of anti-ghosts and multipliers presented in tables 3 and 4, the  $s$ -variations being given by

$$\begin{aligned} sm^{-2} &= \beta^{-1} \quad , \quad s\beta^{-1} = 0 \\ sV_1^{-1} &= b_1 \quad , \quad sb_1 = 0 \\ sn &= \beta^1 \quad , \quad s\beta^1 = 0. \end{aligned} \quad (15)$$

$$\begin{array}{ccccc} & & \psi_2^1 & & \\ & \varphi_1^{-2} & & \varphi_1^2 & \\ \phi^{-3} & & \chi^1 & & \phi^3 \end{array}$$

Table 1: Tower for shift ghosts and anti-ghosts

$$\begin{array}{ccc} & \eta_1^{-1} & \\ \eta^{-2} & & \eta^2 \end{array}$$

Table 2: Tower for shift multipliers

$$\begin{array}{ccccc} & & B_2 & & \\ & V_1^{-1} & & V_1^1 & \\ m^{-2} & & n & & m^2 \end{array}$$

Table 3: Tower for gauge ghosts and anti-ghosts

$$\begin{array}{ccc} & b_1 & \\ \beta^{-1} & & \beta^1 \end{array}$$

Table 4: Tower for gauge multipliers

Thus, the ordinary gauge degrees of freedom of  $B_2$  are fixed by the functional

$$\Sigma_{og} = s \int \{V_1^{-1} d * B_2 + m^{-2} d * V_1^1 + n d * V_1^{-1} - CC\} \quad (16)$$

and the complete gauge-fixed action of the model reads

$$\Sigma = \Sigma_{cl} + \Sigma_Q + \Sigma_{og}. \quad (17)$$

## 2.3 Vector supersymmetry

Due to the fact that we are considering a topological model of Witten-type, one expects the complete gauge-fixed action to admit a VSUSY. At the infinitesimal level, the VSUSY-transformations are described by the operator  $\delta_\tau$  where  $\tau \equiv \tau^\mu \partial_\mu$  is a constant,  $s$ -invariant vector field of ghost-number zero<sup>2</sup>. The variation  $\delta_\tau$  acts as an antiderivation which lowers the ghost-number by one unit and which anticommutes with  $d$ . The operators  $s$  and  $\delta_\tau$  satisfy a graded algebra of Wess-Zumino type,

$$[s, \delta_\tau] = \mathcal{L}_\tau, \quad (18)$$

where  $\mathcal{L}_\tau \equiv [i_\tau, d]$  denotes the Lie derivative along the vector field  $\tau$  and  $i_\tau$  the interior product by  $\tau$ . We will simply refer to the relation (18) as the *SUSY-algebra*.

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<sup>2</sup>In order to avoid technical complications related to the global geometry, we limit the considerations of this section to flat space-time.

The  $\delta_\tau$ -variations of all fields can be determined by applying the general procedure introduced in reference [6]. To start with, we derive the VSUSY-transformations of the classical and ghost fields by expanding the so-called *0-type symmetry conditions*

$$\delta_\tau \tilde{B}_2 = 0 \quad , \quad \delta_\tau \tilde{G}_3 = \mathcal{L}_\tau \tilde{B}_2. \quad (19)$$

with respect to the ghost-number. We thus obtain

$$\begin{aligned} \delta_\tau B_2 &= 0 \quad , \quad \delta_\tau \psi_2^1 = \mathcal{L}_\tau B_2 \\ \delta_\tau V_1^1 &= 0 \quad , \quad \delta_\tau \varphi_1^2 = \mathcal{L}_\tau V_1^1 \\ \delta_\tau m^2 &= 0 \quad , \quad \delta_\tau \phi^3 = \mathcal{L}_\tau m^2. \end{aligned} \quad (20)$$

The  $\delta_\tau$ -variations of the anti-ghosts are found by requiring the  $\delta_\tau$ -invariance of the total action (17) and by applying the commutation relations (18). Finally, the VSUSY-transformations of all multipliers follow from the ones of the corresponding anti-ghosts by imposing the algebra (18) for all of them:

$$\begin{aligned} \delta_\tau \chi_3^{-1} &= 0 \quad , \quad \delta_\tau H_3 = \mathcal{L}_\tau \chi_3^{-1} \\ \delta_\tau \phi^{-3} &= 0 \quad , \quad \delta_\tau \eta^{-2} = \mathcal{L}_\tau \phi^{-3} \\ \delta_\tau \varphi_1^{-2} &= 0 \quad , \quad \delta_\tau \eta_1^{-1} = \mathcal{L}_\tau \varphi_1^{-2} \\ \delta_\tau \chi^1 &= -\mathcal{L}_\tau n \quad , \quad \delta_\tau \eta^2 = \mathcal{L}_\tau \chi^1 + \mathcal{L}_\tau \beta^1 \\ \delta_\tau m^{-2} &= \mathcal{L}_\tau \phi^{-3} \quad , \quad \delta_\tau \beta^{-1} = \mathcal{L}_\tau m^{-2} - \mathcal{L}_\tau \eta^{-2} \\ \delta_\tau V_1^{-1} &= \mathcal{L}_\tau \varphi_1^{-2} \quad , \quad \delta_\tau b_1 = \mathcal{L}_\tau V_1^{-1} - \mathcal{L}_\tau \eta_1^{-1} \\ \delta_\tau n &= 0 \quad , \quad \delta_\tau \beta^1 = \mathcal{L}_\tau n. \end{aligned} \quad (21)$$

Thus, it is by construction that the total action is  $\delta_\tau$ -invariant and that the SUSY-algebra is fulfilled off-shell for all fields of the model. Our results coincide with those found in reference [2] by other methods. We refer to the latter work for the relation of VSUSY to a twist of a supersymmetric field theory.

### 3 Abelian model with Chern-Simons term

The authors of reference [1] considered the interaction of the Abelian 2-forms  $B_2$  and  $B_2^c$  with a non-Abelian Yang-Mills (YM) connection  $A$  by virtue of a Chern-Simons term with coupling constant  $\lambda$ ,

$$\Omega_3(A) = \lambda \operatorname{tr} (AdA + \frac{2}{3}AAA). \quad (22)$$

The proposed action reads<sup>3</sup>

$$\hat{\Sigma}_{cl} = \int (G_3 - \Omega_3) (G_3^c - \Omega_3). \quad (23)$$

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<sup>3</sup>We do not include the ordinary YM-action  $\int \operatorname{tr} (F * F)$  as in reference [1] since it depends on the metric and therefore destroys the topological nature of the model.

This functional represents a six-dimensional topological version of the expression  $\int \text{tr} (G_3 - \Omega_3) * (G_3 - \Omega_3)$  which appears in the action for ten-dimensional supergravity coupled to super YM [3].

The equations of motion for  $A$  and  $B_2$  (or  $B_2^c$ ) have the form

$$F(G_3 - G_3^c) = 0 \quad \text{and} \quad \text{tr} (FF) = 0,$$

where  $F = dA + \frac{1}{2}[A, A]$  denotes the curvature 2-form associated to  $A$ . The latter equations imply  $F = 0$  (i.e. the same equation of motion as in the three-dimensional Chern-Simons theory).

### 3.1 Symmetries

The action (23) is not anymore invariant under the shift  $\delta B_2 = \psi_2^1$ . However, it is invariant under the YM-gauge transformations

$$\begin{aligned} \delta A &= -Dc \equiv -(dc + [A, c]) \\ \delta B_2 &= \lambda \text{tr} (cdA) = \delta B_2^c, \end{aligned} \tag{24}$$

which leave  $G_3 - \Omega_3$  and  $G_3^c - \Omega_3$  invariant.

The BRST-transformations of  $A$  and of the YM-ghost read

$$sA = -Dc \quad , \quad sc = -\frac{1}{2}[c, c]. \tag{25}$$

They follow from the horizontality condition  $\tilde{F} = F$ , where

$$\tilde{F} = d\tilde{A} + \frac{1}{2}[\tilde{A}, \tilde{A}] \quad , \quad \tilde{A} = A + c. \tag{26}$$

The generalized Chern-Simons form

$$\tilde{\Omega}_3 = \lambda \text{tr} (\tilde{A}d\tilde{A} + \frac{2}{3}\tilde{A}\tilde{A}\tilde{A})$$

can be expanded with respect to the ghost-number,

$$\tilde{\Omega}_3 = \Omega_3 + \Omega_2^1 + \Omega_1^2 + \Omega^3, \tag{27}$$

which provides the well-known solution of the descent equations (e.g. see [8])

$$\begin{aligned} \Omega_3 &= \lambda \text{tr} (AdA + \frac{2}{3}AAA) & , & & s\Omega_3 + d\Omega_2^1 &= 0 \\ \Omega_2^1 &= \lambda \text{tr} (cdA) & , & & s\Omega_2^1 + d\Omega_1^2 &= 0 \\ \Omega_1^2 &= \lambda \text{tr} (-ccA) & , & & s\Omega_1^2 + d\Omega^3 &= 0 \\ \Omega^3 &= \lambda \text{tr} (-\frac{1}{3}ccc) & , & & s\Omega^3 &= 0. \end{aligned} \tag{28}$$

We now use this result to discuss the  $B$ -field sector. The generalized field strength of  $B_2$  is defined as before (i.e.  $\tilde{G}_3 = \tilde{d}\tilde{B}_2$  with  $\tilde{B}_2 = B_2 + V_1^1 + m^2$ ), but the horizontality condition (8) of the free model is now replaced by the *horizontality condition*

$$\tilde{G}_3 = G_3 + \Omega_2^1 + \Omega_1^2 + \Omega^3. \quad (29)$$

Expansion with respect to the ghost-number yields the  $s$ -variations

$$\begin{aligned} sB_2 &= -dV_1^1 + \Omega_2^1 \\ sV_1^1 &= -dm^2 + \Omega_1^2 \\ sm^2 &= \Omega^3, \end{aligned} \quad (30)$$

where the explicit expressions for the  $\Omega_p^q(A, c)$  were given in equations (28). Furthermore, substitution of (29) in  $\tilde{d}\tilde{G}_3 = 0$  leads to  $sG_3 = -d\Omega_2^1$  (and reproduces some of the descent equations (28)).

The BRST-transformations (25) and (30) leave the classical action (23) invariant.

## 3.2 Gauge-fixing

In the YM-sector, the gauge symmetry is fixed in the standard way,

$$\Sigma_{gf}^A = s \int \text{tr} \{ \bar{c}d * A \} = \int \text{tr} \{ bd * A - \bar{c}d * Dc \}, \quad (31)$$

where we made use of a BRST-doublet ( $s\bar{c} = b$ ,  $sb = 0$ ).

In the  $B$ -sector, the local symmetry  $\delta B_2 = -dV_1^1$  is fixed as for the free model, i.e. by introducing the gauge-fixing functional  $\Sigma_{gf}^B \equiv \Sigma_{og}$  given by equation (16).

In summary, the complete action of the interacting model reads  $\Sigma_{int} = \hat{\Sigma}_{cl} + \Sigma_{gf}^B + \Sigma_{gf}^A$ .

## 3.3 VSUSY

Due to the absence of shift-symmetries in the present model, the only possible choice for VSUSY-transformations is given by the so-called  *$\emptyset$ -type symmetry conditions* [6]. However, the derivation of  $\delta_\tau$ -variations for all fields is substantially more complicated in the present case since the SUSY-algebra only closes on-shell. Therefore, we will not further elaborate on this point here [9].

## 4 Non-Abelian model

Consider a YM-connection  $A$  and a 2-form potential  $B_2$ , both with values in a given Lie algebra. The field strength of  $B_2$  is now defined by

$$G_3 = DB_2 \equiv dB_2 + [A, B_2] \quad (32)$$

and it satisfies the second Bianchi identity  $DG_3 = [F, B_2]$ , where  $F = dA + \frac{1}{2}[A, A]$  denotes the YM-curvature.

A natural generalization of the action (1) for the Abelian potentials is given by [4]

$$\Sigma_{cl} = \int \text{tr} \{G_3 G_3^c - F[B_2, B_2^c]\}. \quad (33)$$

Neither  $B_2$  nor  $A$  propagate in this model (very much like  $A$  in four-dimensional topological YM-theory).

### 4.1 Symmetries

Following reference [4], we now spell out all local symmetries of the functional (33). As in the previously discussed models, one considers the generalized gauge fields

$$\tilde{A} = A + c \quad , \quad \tilde{B}_2 = B_2 + V_1^1 + m^2 \quad (34)$$

and the associated generalized field strengths

$$\tilde{F} = d\tilde{A} + \frac{1}{2}[\tilde{A}, \tilde{A}] \quad , \quad \tilde{G}_3 = \tilde{D}\tilde{B}_2 \equiv d\tilde{B}_2 + [\tilde{A}, \tilde{B}_2]. \quad (35)$$

The BRST-transformations in the YM-sector can be summarized by the following *horizontality condition* which involves ghost fields for the shifts of  $A$ :

$$\tilde{F} = F + \psi_1^1 + \varphi^2. \quad (36)$$

From this relation and the generalized Bianchi identity  $\tilde{D}\tilde{F} = 0$ , we obtain

$$\begin{aligned} sA &= \psi_1^1 - Dc & , & & s\psi_1^1 &= -D\varphi^2 - [c, \psi_1^1] \\ sc &= \varphi^2 - \frac{1}{2}[c, c] & , & & s\varphi^2 &= -[c, \varphi^2] \end{aligned} \quad (37)$$

and  $sF = -D\psi_1^1 - [c, F]$ .

In the  $B$ -sector, the  $s$ -variations follow from the *horizontality condition* [4]

$$\tilde{G}_3 = G_3 + \psi_2^1 + \varphi_1^2 + \phi^3 \quad (38)$$

and the generalized Bianchi identity  $\tilde{D}\tilde{G}_3 = [\tilde{F}, \tilde{B}_2]$ : they read

$$\begin{aligned} sB_2 &= \psi_2^1 - DV_1^1 - [c, B_2] & , & & s\psi_2^1 &= -D\varphi_1^2 - [c, \psi_2^1] + [F, m^2] + [\psi_1^1, V_1^1] + [\varphi^2, B_2] \\ sV_1^1 &= \varphi_1^2 - Dm^2 - [c, V_1^1] & , & & s\varphi_1^2 &= -D\phi^3 - [c, \varphi_1^2] + [\psi_1^1, m^2] + [\varphi^2, V_1^1] \\ sm^2 &= \phi^3 - [c, m^2] & , & & s\phi^3 &= -[c, \phi^3] + [\varphi^2, m^2] \end{aligned} \quad (39)$$

and  $sG_3 = -D\psi_2^1 - [c, G_3] + [F, V_1^1] + [\psi_1^1, B_2]$ . The action (33) is inert under the BRST-transformations (37),(39) which are nilpotent by construction.

## 4.2 Gauge-fixing

The shift- and ordinary gauge symmetry in the  $B$ -sector are fixed as in the free Abelian model, except that all fields are now Lie algebra-valued. Thus, the total gauge-fixing action in the  $B$ -sector is given by

$$\Sigma^B = \Sigma_Q^B + \Sigma_{og}^B, \quad (40)$$

with

$$\begin{aligned} \Sigma_Q^B &= s \int \text{tr} \left\{ \chi_3^{-1} (*G_3 + G_3^c) + [\varphi_1^{-2} d * \psi_2^1 + \phi^{-3} d * \varphi_1^2 + \chi^1 d * \varphi_1^{-2} - CC] \right\} \\ \Sigma_{og}^B &= s \int \text{tr} \left\{ V_1^{-1} d * B_2 + m^{-2} d * V_1^1 + nd * V_1^{-1} - CC \right\}, \end{aligned} \quad (41)$$

The BRST-transformations of the anti-ghost and multiplier fields are given by equations (11),(13) and (15).

In the YM-sector, the gauge-fixing can be done along the lines of four-dimensional topological YM-theory. However, the familiar four-dimensional self-duality condition for the curvature form  $F$  does not make sense in six dimensions and it has to be generalized by introducing a 4-form  $T_4$  which is invariant under some maximal subgroup of  $SO(6)$  [10, 11]: the self-duality condition can then be written as

$$*F = \Omega_2 F \quad \text{with} \quad \Omega_2 \equiv *T_4. \quad (42)$$

This constraint is implemented by the gauge-fixing action

$$\Sigma_{sd}^A = s \int \text{tr} \left\{ \chi_4^{-1} (*F - \Omega_2 F) \right\}, \quad (43)$$

where  $\chi_4^{-1}$  belongs to a BRST-doublet ( $s\chi_4^{-1} = H_4$ ,  $sH_4 = 0$ ). The residual gauge symmetries can then be fixed as in topological YM-theory by using a linear gauge-fixing term [12, 6],

$$\Sigma^A = \Sigma_{sd}^A + s \int \text{tr} \left\{ \bar{\phi}^{-2} d * \psi_1^1 + \bar{c} d * A \right\}, \quad (44)$$

which involves the BRST-doublets  $(\bar{\phi}^{-2}, \eta^{-1})$  and  $(\bar{c}, b)$ . In summary, the total gauge-fixed action is given by  $\Sigma = \Sigma_{cl} + \Sigma^B + \Sigma^A$ .

### 4.3 VSUSY

In order to derive the VSUSY-transformations, one considers 0-*type symmetry conditions* in the  $A$ - and  $B$ -sectors:

$$\begin{aligned} \delta_\tau \tilde{A} &= 0 & , & & \delta_\tau \tilde{F} &= \mathcal{L}_\tau \tilde{A} \\ \delta_\tau \tilde{B}_2 &= 0 & , & & \delta_\tau \tilde{G}_3 &= \mathcal{L}_\tau \tilde{B}_2. \end{aligned} \quad (45)$$

By expanding with respect to the ghost-number and substituting the horizontality conditions (36),(38), one finds

$$\delta_\tau(A, c) = 0 \quad , \quad \delta_\tau(B_2, V_1^1, m^2) = 0. \quad (46)$$

and

$$\begin{aligned} \delta_\tau \psi_1^1 &= \mathcal{L}_\tau A & , & & \delta_\tau \psi_2^1 &= \mathcal{L}_\tau B_2 \\ \delta_\tau \varphi^2 &= \mathcal{L}_\tau c & , & & \delta_\tau \varphi_1^2 &= \mathcal{L}_\tau V_1^1 \\ & & & & \delta_\tau \phi^3 &= \mathcal{L}_\tau m^2. \end{aligned} \quad (47)$$

The  $\delta_\tau$ -variations for the BRST-doublets occurring in the gauge-fixing action of the  $B$ -sector are given by equations (21) and those in the YM-sector read

$$\begin{aligned} \delta_\tau \bar{c} &= -\mathcal{L}_\tau \bar{\phi}^{-2} & , & & \delta_\tau b &= \mathcal{L}_\tau \bar{c} + \mathcal{L}_\tau \eta^{-1} \\ \delta_\tau \chi_4^{-1} &= 0 & , & & \delta_\tau H_4 &= \mathcal{L}_\tau \chi_4^{-1} \\ \delta_\tau \bar{\phi}^{-2} &= 0 & , & & \delta_\tau \eta^{-1} &= \mathcal{L}_\tau \bar{\phi}^{-2}. \end{aligned} \quad (48)$$

The total action is invariant under the given VSUSY-transformations which satisfy the VSUSY-algebra off-shell.

## 5 Concluding remarks

A possible generalization of the non-Abelian model consists of the addition of a  $BF$ -term  $\int \text{tr}(B_4 F)$  (see [8] and references therein for such models in arbitrary dimensions): such a term breaks the invariance under shifts of  $A$ . Other possible extensions [4] are the inclusion of the topological invariant  $\int \Omega_2 \text{tr}(FF)$  (where  $\Omega_2$  is a closed 2-form), which leads to “nearly topological” field theories [11], or the addition of a term  $\int \text{tr}(F DZ_3)$  involving a 3-form  $Z_3$ . The resulting models can be discussed along the lines of the present paper [9].

## References

- [1] L. Baulieu and P. West, “Six-dimensional TQFTs and twisted supersymmetry,” *Phys. Lett.* **B436** (1998) 97, hep-th/9805200.

- [2] H. Ita, K. Landsteiner, T. Pisar, J. Rant, and M. Schweda, “Remarks on topological SUSY in six-dimensional TQFTs,” *JHEP* **11** (1999) 35, [hep-th/9909166](#).
- [3] G.F. Chapline and N.S. Manton, “Unification of Yang-Mills theory and supergravity in ten dimensions, *Phys.Lett.* **120B** (1983) 105;  
M. Green, J. Schwarz and E. Witten, *Superstring Theory Vol. 2* (Cambridge University Press, 1997).
- [4] L. Baulieu, “On forms with non-Abelian charges and their dualities,” *Phys. Lett.* **B441** (1998) 250, [hep-th/9808055](#).
- [5] R. A. Bertlmann, *Anomalies in quantum field theory* (Clarendon Press, Oxford 1996).
- [6] F. Gieres, J. Grimstrup, T. Pisar and M. Schweda, “Vector supersymmetry in topological field theories”, [hep-th/0002167](#).
- [7] E. Witten, “Topological quantum field theory,” *Commun. Math. Phys.* **117** (1988) 353;  
L. Baulieu and I. M. Singer, “Topological Yang-Mills Symmetry,” *Nucl. Phys.B (Proc.Suppl.)* **5B** (1988) 12;  
D. Birmingham, M. Rakowski and G. Thompson, “Topological field theory”, *Phys.Rep.* **209** (1991) 129.
- [8] O. Piguet and S. P. Sorella, *Algebraic Renormalization* (Springer Verlag, 1995).
- [9] T. Pisar, *Supersymmetric structures in topological field models* (Ph.D. Thesis, TU Wien, June 2000).
- [10] E. Corrigan, C. Devchand, D.B. Fairlie and J. Nuyts, “First-order equations for gauge fields in spaces of dimension greater than four”, *Nucl.Phys.* **B214** (1983) 452;  
H. Kanno, “A note on higher dimensional instantons and supersymmetric cycles”, *Prog.Theor.Phys.Suppl.* **135** (1999) 18.
- [11] L. Baulieu, H. Kanno and I.M. Singer, “Special quantum field theories in eight and other dimensions”, *Commun.Math.Phys.* **194** (1998) 149, [hep-th/9704167](#).
- [12] A. Brandhuber, O. Moritsch, M. W. de Oliveira, O. Piguet, and M. Schweda, “A Renormalized supersymmetry in the topological Yang-Mills field theory,” *Nucl. Phys.* **B431** (1994) 173–190, [hep-th/9407105](#).